Asteroseismic modelling of the solar-like star β Hydri

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Abstract We present the results of modelling the subgiant star β Hydri using the seismic observational constraints. We have computed several grids of stellar evolutionary tracks using Aarhus STellar Evolution Code (ASTEC, Christensen-Dalsgaard, 2008a), with and without helium diffusion and settling. For those models on each track that are located at the observationally determined position of β Hydri in the HR diagram, we have calculated the oscillation frequencies using Aarhus adiabatic pulsation package (ADIPLS, Christensen-Dalsgaard, 2008b). Applying the near-surface corrections to the calculated frequencies using the empirical law presented by Kjeldsen et al. (2008), we have compared the corrected model frequencies with the observed frequencies of the star. We show that after correcting the frequencies for the near-surface effects, we have a fairly good fit for both $l=0$ and $l=2$ frequencies. We also have good agreement between the observed and calculated $l=1$ mode frequencies although there is room for improvement in order to fit all the observed mixed modes simultaneously.

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1 Introduction

β Hydri is a G2-type subgiant star exhibiting solar like oscillations. It is often referred to as the future of the Sun due to having parameters (Table 1) close to those of the Sun, in addition to being at a later evolutionary stage than the Sun. Being rather evolved, β Hydri exhibits mixed modes in its observed spectrum. This makes the star particularly interesting for asteroseismic studies as mixed modes carry more information about the core than do the regular p-mode frequencies.

Table 1 Parameters of β Hydri as given in literature. The radius was derived from the interferometric angular diameter (North et al. 2007) and the revised Hipparcos parallax (van Leeuwen 2007). The mass was then derived from the mean density from asteroseismology.

Parameter	Value	Reference
$M/M_\odot$	1.085±0.028	Kjeldsen et al. 2008
$R/R_\odot$	1.809±0.015	Kjeldsen et al. 2008
$L/L_\odot$	3.494±0.087	Current work
$T_{\rm eff}$(K)	5872±44	North et al. 2007
[Fe/H]	-0.08±0.04	Santos et al. 2005
$\Delta\nu_0(\mu{\rm Hz})$	57.24±0.16	Bedding et al. 2007

An extensive analysis of this star was done by Fernandes & Monteiro (2003), who emphasized the possibility of employing seismic constraints to remove partially the parameter degeneracy that exists when only the non-seismic observational constraints (such as $T_{\rm eff}$ and luminosity) are used. Those constraints alone are not enough, as one can obtain the same location of a stellar model in the Hertzsprung-Russell (HR) diagram using different stellar parameters (such as mass, X, Y,

Z, etc.). Fernandes & Monteiro (2003) used the large frequency separation, $\Delta\nu_0$, obtained by Bedding et al. (2001), to show how to constrain the mass independently and also note the need for individual frequencies to further constrain the age of the star using the characteristics of the frequency spectrum that are related to the stellar core, such as mixed modes.

The first preliminary comparison of the individual observed frequencies and model frequencies was done by Di Mauro et al. (2003). They presented a model with large and small frequency separation that are within the limits derived from the observations (Bedding et al. 2001). They showed that the match between the model and observed frequencies are satisfactory except for the $l=1$ modes, some of which are affected by avoided crossings. This emphasizes again the importance of accurate asteroseismic observations and detailed analysis in order to evaluate the stellar interiors.

Here, we use the latest asteroseismic observational constraints (Bedding et al. 2007) consisting of individual frequencies including some modes which are identified to be possible mixed modes. We present the methods to search for a best model and the resulting best fits within the parameter space of our survey.

2 Methods

We started calculating the grids of evolutionary tracks with a wide range of parameters and large steps of increment. Analyzing the first grid we have selected the best models, around which we have computed denser grids. The initial parameters of the grids are given in Table 2. Diffusion and gravitational settling of helium are added in the third grid, in which not all the tracks have been successfully completed. We have carried on our analysis with the tracks that did not have convergence problems.

On each track in the grids, we have selected the models having parameters within the observational uncertainty limits. We have calculated the oscillation frequencies of those models and compared with the observations. For the purpose of comparison we used the frequencies resulting from dual-site radial velocity observations with HARPS and UCLES spectrographs (Bedding et al. 2007).

Before comparing the calculated frequencies with the observed ones we have applied near-surface corrections to the calculated frequencies. The correction is needed due to the fact that existing stellar models fail to represent properly the near-surface layers of the solar-like stars, where the turbulent convection takes place. This affects the high frequencies most; thus the correction should be applied in a way that low frequencies are much less affected.

The situation is the same for the Sun, and the difference between observed and calculated frequencies is shown to be well approximated by the empirical power law given by Kjeldsen et al. (2008) as

$$\nu_{\text{obs}}(n) - \nu_{\text{best}}(n) = a \left[\frac{\nu_{\text{obs}}(n)}{\nu_0} \right]^b, \tag{1}$$

where ν_{obs} are the observed $l=0$ frequencies with radial order n, ν_{best} are the corresponding calculated frequencies of the best model, and ν_0 is a constant frequency chosen to be the frequency corresponding to the peak power in the spectrum, which is taken as 1000μHz for β Hydri. Kjeldsen et al. (2008) used the solar data and models to calibrate the exponent b, which is calculated as 4.90 for the Sun. Using this solar b value, and calculating a for each model, we have applied the near-surface corrections to the model frequencies. The corrections for mixed modes are probably less than for pure p modes, as discussed by Kjeldsen et al. (2008), and so we have set them to zero. We have then selected the best models performing a chi-square minimization test for the goodness of the fit as

$$\chi^2 = \frac{1}{N} \sum_{n,l} \left(\frac{\nu_l^{\text{model}}(n) - \nu_l^{\text{obs}}(n)}{\sigma(\nu_l^{\text{obs}}(n))} \right)^2, \tag{2}$$

where N is the total number of modes included, $\nu_l^{\text{obs}}(n)$, and $\nu_l^{\text{model}}(n)$ are the observed frequencies, and the corrected model frequencies, respectively, for each spherical degree l and the radial order n, and σ represents the uncertainty in the observed frequencies.

3 Results

Properties of our best models are given in Table 3, with the corresponding so-called échelle diagrams in Figures 1 and 2 (see Christensen-Dalsgaard (2004) and references therein for the explanation of an échelle diagram). The increase in the systematic difference between the observed and model frequencies with increasing frequency can be seen in the left panels of the échelle diagrams.
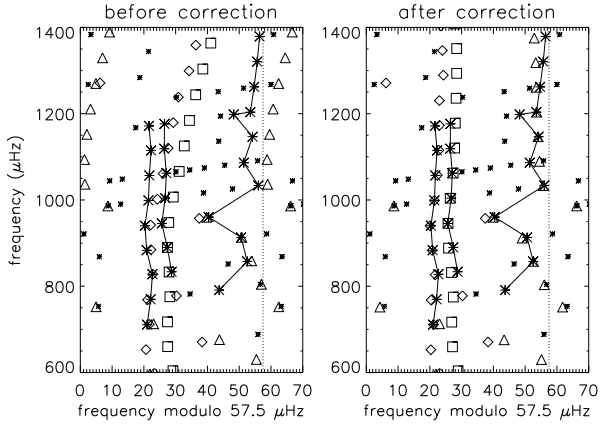
The échelle diagrams include all the observed frequencies, while those having identified modes are connected by a solid line for clarity. The observed frequencies which are not connected by a line and shown with smaller symbols are the ones that are not assigned a mode by the observers (Bedding et al. 2007); however, they note that those frequencies will include some genuine modes although some of them might be sidelobes

Table 2 Parameters used to compute the evolutionary tracks

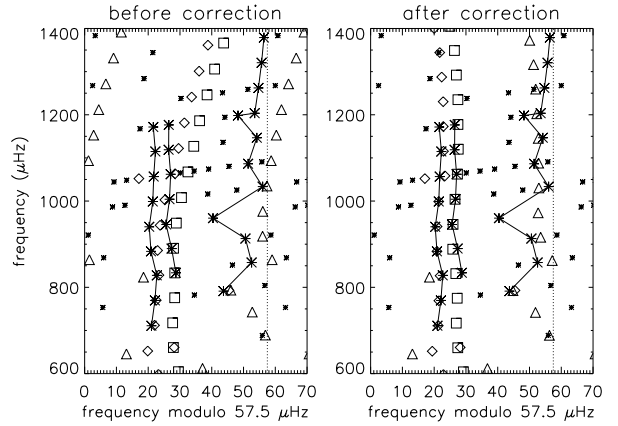| Parameter	Grid 1	Grid 2	Grid 3
$M/M_\odot$	1.04–1.10 (with steps of 0.01)	1.076–1.084 (with steps of 0.002)	1.076–1.084 (with steps of 0.002)
Z/X	0.018–0.022 (with steps of 0.001)	0.018–0.022 (with steps of 0.001)	0.018–0.022 (with steps of 0.001)
Y	0.23, 0.27, 0.28, 0.30	0.276–0.284 (with steps of 0.002)	0.276–0.284 (with steps of 0.002)
Mixing length parameter (α)	1.4–1.8 (with steps of 0.2)	1.75–1.85 (with steps of 0.025)	1.75–1.85 (with steps of 0.025)
Diffusion & gravitational settling	None	None	He

Table 3 Parameters of the best models

Parameter	From grid 2	From grid 3
$M/M_\odot$	1.082	1.082
Z	0.01346	0.01266
Y	0.278	0.284
α	1.825	1.775
Age (Gyr)	6.447	5.712
$R/R_\odot$	1.814	1.806
$L/L_\odot$	3.444	3.473
T_{eff}(K)	5844	5869
a	-2.76	-3.76
χ^2	7.71	11.93

Fig. 1 Échelle diagram of the best model without gravitational settling or diffusion. Left (right) panel shows the case before (after) applying near-surface corrections. Stars denote the observations, squares correspond to the model frequencies with $l=0$, triangles to the model frequencies with $l=1$, and diamonds to the model frequencies with $l=2$. Dotted vertical lines indicate the value of the large frequency separation, $\Delta\nu_0$

or noise peaks. The size of the largest symbols cover approximately 1.5-σ uncertainty to both sides on the horizontal axis. Three of the identified $l=1$ frequencies that fall to the left side of the $l=1$ ridge are identified as mixed modes by the observers (Bedding et al. 2007). The model without He settling or diffusion reproduces the observed frequencies very well except for the lowest

mixed mode. It is seen from Fig.1 that some of the unidentified observed frequencies also match the model frequencies quite well. When the diffusion and settling

Fig. 2 Échelle diagram of the best model with He settling and diffusion. The symbols are used in the same way as in Fig. 1

of helium is added, the agreement between the model and the observations is less strong than the previous case. Although the lowest mixed mode is reproduced in this model, the mixed mode having the sharpest character (largest departure from the $l=1$ ridge) is not. The highest mixed mode has not been reproduced by any model frequency with $l=1$, suggesting that $l=3$ frequencies might be investigated to search for a match to that frequency. The positions of both models on the subgiant branch are shown in Fig. 3. The model with diffusion is younger due to having completed the main sequence phase faster, as the hydrogen mass fraction decreases faster owing to diffusion and settling bringing helium to the core. Furthermore, the hook shape in the evolutionary track with diffusion is due to the fact that the star had grown a convective core up to an extent where it contained almost 4% of the stellar mass.
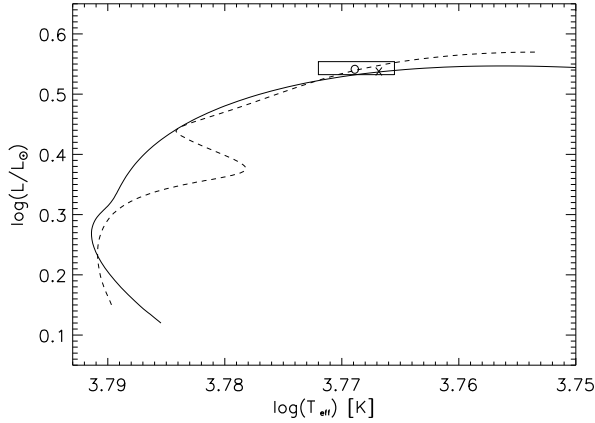
Fig. 3 Evolutionary tracks with He settling and diffusion (dashed curve, the best model marked with a circle) and without (solid curve, the best model marked with a cross)

4 Conclusions

Our results justify that the empirical power law representing the effect of near-surface layers in the Sun works for β Hydri as well. Our best models with and without He settling and diffusion reproduce the observed $l=0$ and $l=2$ modes well; however, the fit at $l=1$ modes is relatively poor due to the mixed modes, but still satisfactory. It is important to investigate the possibility of any of the observed modes being an $l=3$ mode.

Furthermore, our results are in agreement with the findings of Fernandes & Monteiro (2003), who derived, through the HR diagram analysis, the mass to be $1.10^{+0.04}_{-0.07}$ $M_\odot$, the helium abundance to be between 0.25 and 0.30, and the stellar age to be between 6.4 and 7.1 Gyr. We, however, note that the age of our model with helium diffusion is less than the lower limit of the cited result. Employing $\Delta\nu_0$, they found the mass to be 1.09 ± 0.22 $M_\odot$, noting that the large uncertainty is to be improved with the improved accuracy of the observations. Further analysis may be carried out to investigate the effect of convective core overshooting as our models had convective cores at some earlier stage in their evolution.

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